Deriving the quark condensate within a finite Fermi system from the generating functional of chiral perturbation theory

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Abstract

The generating functional of heavy baryon chiral perturbation theory at order $\mathcal{O}(Q^2)$ in the mean field approximation (with a pseudoscalar source coupling which is consistent with the PCAC-Ward identities on the current quark level) has been exploited to derive Migdal's in–medium pion propagator. It is shown that the prediction for the density dependence of the quark condensate obtained on the composite hadron level by embedding PCAC within the framework of Migdal's approach to finite Fermi systems is identical to that resulting from QCD.

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I. INTRODUCTION

The evolution of the quark condensate, $\langle\, 0 \mid \bar{q}q \mid 0\,\rangle$, with density, ρ, is one of the most intriguing and controversial problems of intermediate hadron physics that had attracted much attention during the last years partly because of its relevance for chiral symmetry restoration at finite densities. On the one hand, QCD–inspired quark models predict an in fact model independent linear decrease of the quark condensate in accordance with $\langle \widetilde{0} \mid \bar{q}q \mid \widetilde{0}\rangle = \langle 0 \mid \bar{q}q \mid 0\rangle(1 - \sigma_N\rho/f_\pi^2 m_\pi^2)$ [1–4]. Here m_π and f_π in turn denote the mass and weak decay constant of the pion, whereas σ_N stands for the pion–nucleon sigma term. In the following "tilde" will be used to denote in–medium states. On the other hand, attempts have been done to deduce the properties of quark matter at finite densities from the properties of composite hadrons at finite densities [5,6] with the aim to extend the linear decrease of $\langle \widetilde{0} \mid \bar{q}q \mid \widetilde{0}\rangle$ with ρ (which was mentioned above) to higher powers of the matter density. The scheme exploited is based on the evaluation of the Gell–Mann–Oakes–Renner (GOR) relation (see [7] for a recent review). The latter relates the quark condensate to the divergence of the charged axial vector current and can be evaluated in exploiting PCAC in combination with Migdal's pion propagator. The basic ingredient of such schemes was the assumption on the linear dependence of the S–wave pion self–energy on matter density. We here show that if Migdal's propagator is derived from the generating functional of heavy baryon chiral perturbation theory at order $\mathcal{O}(Q^2)$ in the mean field approximation, the S–wave pion self–energy is no longer linear in ρ but contains an infinite number of higher powers of matter density. With that the prediction for the density dependence of the quark condensate obtained on the composite hadron level by embedding PCAC within the framework of Migdal's approach to dense matter is shown to be identical to that resulting from QCD. In this way, the possibility to learn more on the evolution of the quark condensate with density from the in–medium composite hadron physics is ruled out. Through the pseudoscalar source coupling used by us consistency is ensured between PCAC on the composite hadron level, on the one side, and PCAC-Ward

identities on the current quark level, on the other side.

The paper is organized as follows. In the next section we review shortly the basic ingredients of chiral perturbation theory in the heavy baryon limit and in the mean field approximation, present the corresponding effective chiral lagrangian, and derive Migdal's pion propagator. In sect. 3 we evaluate the GOR relation within a finite Fermi system before closing with a short summary and discussion.

II. DERIVING MIGDAL'S PION PROPAGATOR FROM A CHIRAL EFFECTIVE LAGRANGIAN

Chiral perturbation theory [8] was developed as a tool for constructing Green functions in QCD. It is based on the assumption that the fundamental symmetry of QCD, the chiral symmetry, is realized in the Nambu–Goldstone mode with the pions acting as Goldstone bosons. As long as Goldstone particles interact only weakly with each other and the matter (fermion) fields, it is possible to expand correlation functions in powers of the light quark masses and the external pion momenta thought to be very small on the hadronic scale of $\Lambda \approx 1GeV$. An $SU(3)_L \otimes SU(3)_R$ chiral effective lagrangian (subsequently denoted by $\mathcal{L}_{\rm MG}$) containing the characteristic momenta to second order (so called "next–to–leading" order) was constructed by Manohar and Georgi [9] in the heavy-baryon formalism or static approximation. In the $SU(2)_L\otimes SU(2)_R$ reduction and in the S–channel the latter lagrangian can be expanded in the pion field and cast into the following form [10,11] which is valid to order $\mathcal{O}(Q^2)$ with Q standing for the external pion momentum or mass:

$$\begin{aligned}\mathcal{L}_{\rm MG} &= i\bar{N}(x)(v\cdot\partial)N(x) - \sigma\bar{N}(x)N(x) + \frac{1}{2}\left(\partial\,\pi(x)\right)^2 - \frac{1}{2}m_\pi^2\,\pi(x)^2 \\ &+ \frac{1}{f_\pi^2}\left(\frac{1}{2}\sigma\pi(x)^2 + c_2(v\cdot\partial\pi(x))^2 + c_3(\partial\pi(x))^2\right)\bar{N}(x)N(x) + \cdots\,. \end{aligned} \tag{1}$$

The quantity v_μ stands for the four-velocity of the nucleon (N) in the heavy baryon limit, and reduces to $v_\mu = (1,0,0,0)$ for a nucleon at rest, whereas $\pi^a(x)$ stands for the pion field at zero matter density. The constant coefficient σ is linear in the quark masses and

therefore of the order $\mathcal{O}(Q^2)$. It serves to *increase* the nucleon mass over its $SU(2)$ chiral limit value of $m_0 = 890 MeV$ to $m_N = m_0 + \sigma > m_0$. By this requirement the sign of σ is fixed to be positive. Thus modulo $\mathcal{O}(Q^3)$ corrections, the identification $\sigma = \sigma_N$, where σ_N is the pion–nucleon–sigma term, is natural [12]. The combination $(c_2 + c_3)m_\pi^2$ can then be extracted (on the tree level) from the empirical isospin even pion nucleon scattering length $a_{\pi N}^+$ as $(c_2+c_3)m_\pi^2 \approx -26\,\mathrm{MeV}$ [10]. Note, the coefficients c_2 and c_3 differ from those defined in ref. [10] by $\mathcal{O}(Q^3)$ terms. The relevant generating functional at the order $\mathcal{O}(Q^2)$ reads [8,13,14]

$$e^{iZ_{\rm eff}[s,p,V_\mu,A_\mu,\eta,\bar\eta]} = \mathcal{N} \int dU dN d\bar{N} \, \exp\left(i \int d^4x \, \left\{ \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \bar\eta N + \bar N \eta \right\} \right) . \tag{2}$$

Here, $\mathcal{N}$ is an overall normalization factor, $U, N, \bar N$ are dummy integration fields for the pions in the non-linear representation and the nucleons in the heavy baryon formulation, respectively. The lagrangians $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi N}$ are defined in such a way that they do not only depend on the fields, but also on the scalar (s), pseudoscalar (p), vector (V_μ) and axial vector (A_μ) sources. The sources $\bar\eta$ and η generate one–nucleon in– and out–states, respectively. The lagrangian entering the generating functional (2), $\mathcal{L}_{\rm GSS} := \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N}$ (which we denote the lagrangian of Gasser, Sainio and Švarc [13]), is given by the nucleon kinetic energy term (which is of leading order, $\mathcal{O}(Q)$) and to subleading order, $\mathcal{O}(Q^2)$, by [10,14]

$$\begin{aligned}
\mathcal{L}_{\pi\pi}^{(2)} &= \frac{f_\pi^2}{4} \mathrm{Tr}\left((\nabla^\mu U)^\dagger \nabla_\mu U \right) + \frac{f_\pi^2}{4} \mathrm{Tr}(U^\dagger \chi + \chi^\dagger U) \,, \\
\mathcal{L}_{\pi N}^{(2)} &= c_2 \bar N \left(i u^\dagger \nabla_\mu U u^\dagger \, v^\mu \right)^2 N + c_3 \bar N \left(i u^\dagger \nabla_\mu U u^\dagger \right)^2 N - \frac{\sigma}{4 m_\pi^2} \bar N N \, \mathrm{Tr}(U^\dagger \chi + \chi^\dagger U) \,, \\
\nabla_\mu U &:= \partial_\mu U - i \frac{1}{2} \left\{ \tau^a A_\mu^a, U \right\} - i \frac{1}{2} \left[\tau^a V_\mu^a, U \right] \,, \\
\chi &:= 2B(s + i\tau^a p^a) = 2 \frac{m_\pi^2}{m_u + m_d} (s + i \tau^a p^a) \,,
\end{aligned}$$

where $U = u^2 = \exp(i\tau^a \pi^a / f_\pi)$ and m_u and m_d are the up– and down-quark masses, respectively. Note that the sources are coupled in such a way that the lagrangian (including the sources) is in fact even *locally* chiral invariant [8]. Setting the vector and axial vector sources, V_μ and A_μ, to zero, using the scalar source s to generate the quark mass matrix

s=$\mathcal{M}$=diag(m_u, m_d) and finally expanding U to second order in π^a, one gets the following relation between the $\mathcal{L}_{\rm GSS}$ lagrangian and $\mathcal{L}_{\rm MG}$ of (1):

$$\mathcal{L}_{GSS}|_{A_\mu=V_\mu=0,s=\mathcal{M}} = \mathcal{L}_{\rm MG} + \left(1 - \frac{\sigma_N \bar{N}(x)N(x)}{f_\pi^2 m_\pi^2}\right) j^a(x)\pi^a(x) \ . \tag{3}$$

Here, we introduce a "renormalized" pseudoscalar isovector j^a which is related to the original source p^a by $j^a(x) = 2Bf_\pi \, p^a(x) = g_\pi p^a(x)$ where g_π is the vacuum coupling constant of the pseudoscalar density to the pion [8], $\langle\, 0 \mid \bar{q} i\gamma_5 \tau^a q \mid \pi^b \,\rangle = \delta^{ab} g_\pi$. Note that via the generating functional formalism of [8] (where the symmetry-breaking quark–mass terms are accounted for by the scalar source s) the chirally symmetric structure of the source couplings at the QCD level (which is even *local* if the axial vector and vector source transform covariantly) is manifestly kept at the effective lagrangian level – in case all sources are non–zero, of course. Thus the pseudoscalar source p^a at the effective lagrangian level is to be identified at the QCD level with the source which directly couples to the pseudoscalar quark density. Therefore the PCAC-Ward identities between the amplitudes involving on the one hand the axial current and on the other hand the pseudoscalar density are kept unaltered in the framework of the effective lagrangian $\mathcal{L}_{\rm GSS}$. This lagrangian will automatically generate the PCAC-consistent amplitudes, e.g. the isospin-symmetric S–wave πN amplitudes at the Weinberg, Adler, Cheng-Dashen point and at threshold, see ref. [14]. All the measurable quantities, however, are of course independent of the special choice of the pseudoscalar source coupling (see ref. [15]) and will follow from $\mathcal{L}_{\rm GSS}$ as well as $\mathcal{L}_{\rm MG}$ or any other variant based on $\mathcal{L}_{\rm MG}$ [14].

Now in the mean field limit, where $\bar{N}(x)N(x)$ (which is equal to $N(x)^\dagger N(x)$ in the heavy-baryon formulation) is approximated by the local matter density $\rho(x)$, the chiral lagrangian $\mathcal{L}_{\rm GSS}$ if rewritten in terms of the U field takes the manifestly chirally symmetric form (see [14], [16] for details) :

$$\mathcal{L}_{\rm MF} = \frac{f_\pi^2}{4}(g^{\mu\nu} + \frac{D^{\mu\nu}\rho}{f_\pi^2})\,{\rm Tr}\left((\nabla_\mu U)^\dagger \nabla_\nu U\right) + \frac{f_\pi^2}{4}(1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2})\,{\rm Tr}(U^\dagger \chi + \chi^\dagger U) + \cdots \ , \tag{4}$$

with $D^{\mu\nu} := 2c_2 v^\mu v^\nu + 2c_3 g^{\mu\nu}$. The weak axial vector decay constant of the in-medium pion

is the time–like one because of the special case of the S–channel considered and is identified from the first term in eq. (4) as

$$\left(f_\pi^t(\rho)\right)^2 = f_\pi^2\left(1+\frac{2(c_2+c_3)\rho}{f_\pi^2}\right) + \mathcal{O}(m_\pi)\,. \tag{5}$$

More precisely, it is extracted from the residuum of time-like axial current–current two-point function

$$\frac{\delta}{\delta A_0^a(-q)}\frac{\delta}{\delta A_0^b(q)}\,Z_{\rm MF}|_{A_\mu=V_\mu=p=0,s=\mathcal{M}}$$

where $Z_{\rm MF}$ is the to $Z_{\rm eff}$ analogous generating functional in the mean-field approximation — see ref. [14] for more details. In complete analogy, the in–medium condensate can be read off from the second term of eq. (4) as follows:

$$\langle\,\widetilde{0}\mid\bar{q}q\mid\widetilde{0}\,\rangle = \langle\,0\mid\bar{q}q\mid 0\,\rangle\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right) + \mathcal{O}(m_\pi)\,. \tag{6}$$

Here, the vacuum quark condensate is given by $\langle\,0\mid\bar{q}q\mid 0\,\rangle = -2f_\pi^2 B + \mathcal{O}(m_\pi^2)$. Again, the precise derivation is based on the generating functional techniques:

$$\frac{\delta}{\delta s(x)}Z_{\rm MF}|_{A_\mu=V_\mu=p=s=0} = -\langle\,\widetilde{0}\mid\bar{u}u+\bar{d}d\mid\widetilde{0}\,\rangle$$

(see ref. [14]). The propagator of a charged pion in the medium is developed from the mean-field analog of the generating functional (2) in the standard way as

$$\frac{\delta}{\delta j^a(q)}\frac{\delta}{\delta j^b(-q)}Z_{\rm MF}|_{A_\mu=V_\mu=j=0,s=\mathcal{M}} = \Delta_{\Phi_\pi}(q^2,\rho)\,\delta_{ab}\,,$$

with

$$\Delta_{\Phi_\pi}(q^2,\rho) = \frac{(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2})^2}{q_0^2(1+\frac{2(c_2+c_3)\rho}{f_\pi^2}) - \vec{q}^{\,2}(1-\frac{2c_3\rho}{f_\pi^2}) - m_\pi^2(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2})} + \mathcal{O}(m_\pi^3) \tag{7}$$

$$= \frac{\left(g_\pi^*(\rho)/g_\pi\right)^2}{q_0^2 - \vec{q}^{\,2}\frac{1-2c_3\rho/f_\pi^2}{1+2(c_2+c_3)\rho/f_\pi^2} - {m_\pi^*}^2} + \mathcal{O}(m_\pi^3)\,, \tag{8}$$

where $g_\pi^*(\rho)$ is the in-medium coupling constant of the pseudoscalar density to the pion [14]

$$(g_\pi^*(\rho))^2 = g_\pi^2\frac{\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)^2}{1+\frac{2(c_2+c_3)\rho}{f_\pi^2}}\,. \tag{9}$$

The in–medium pion mass m_π^* is determined by the pole of this propagator and is given by (see refs. [5,14])

$$(m_\pi^*)^2 = m_\pi^2 \, \frac{1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2}}{1 + \frac{2(c_2+c_3)\rho}{f_\pi^2}} \approx m_\pi^2 \, . \tag{10}$$

From eqs. (5–6) (with $m_\pi^2 = B(m_u+m_d)+\mathcal{O}(m_\pi^4)$) and eq. (10) the explicit in–medium extension of the Gell–Mann–Oakes–Renner relation immediately follows as

$$\begin{aligned} \left(f_\pi^t(\rho)\right)^2 (m_\pi^*)^2 &= -2m_q \langle \widetilde{0} \mid \bar{q}q \mid \widetilde{0} \rangle + \mathcal{O}(m_\pi^3) \\ &= f_\pi^2 m_\pi^2 \left(1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2}\right) + \mathcal{O}(m_\pi^3) \, , \end{aligned} \tag{11}$$

where m_q stands for the averaged quark mass [14]. Finally, the PCAC-consistent pion field at finite density is related to the pion field at zero density according to

$$\widetilde{\pi}^a(\rho) = \frac{\delta Z_{\rm MF}[j^a, \rho]}{\delta j^a} = (1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2})\pi^a \, . \tag{12}$$

Whereas the denominator of the propagator (7) and especially its pole position is independent of the GSS-choice for the pseudoscalar source coupling and therefore also of PCAC, the numerator and $g_\pi^*(\rho)$ result from the specific structure of the pseudoscalar source coupling and thus are scheme–dependent, here PCAC–scheme–dependent. Note that in case the effective lagrangian $\mathcal{L}_{\rm GSS}$ were replaced by the lagrangian $\mathcal{L}_{\rm MG}$ plus a trivial nucleon-independent coupling to the pseudoscalar source, $j^a(x)\pi^a(x)$, the term $(1-\sigma_N\rho/f_\pi^2 m_\pi^2)^2$ in the numerator of the propagator (7) and in the definition of $g_\pi^*(\rho)$ in eq. (9) would be absent, indicating that the "MG" choice for the in–medium pion field, $\pi^a_{\rm MG}(\rho) = \pi^a$, is not respecting PCAC. The results of eqs.(5–6) and (10–11) which all refer to measurable quantities do of course not depend on the PCAC choice for the in-medium pion field (12) (as the pseudoscalar source p^a or j^a did not enter in any of the derivations) and would follow from the "MG"-lagrangian as well [14].

Now one can compare the expression (7) with the standard form of Migdal's propagator in the finite Fermi systems theory (FFS) [17]

$$(\Delta_\pi)_{\rm FFS}(q^2, \rho) = \left\{ q_0^2 \left(1 - \frac{\partial \Pi^S}{\partial q_0^2}\right) - \gamma(\rho)\vec{q}^{\,2} - m_\pi^2 \left(1 + \frac{\Pi^S(0,0)}{m_\pi^2}\right) \right\}^{-1} \, , \tag{13}$$

where the S–wave pion self energy $\Pi^S(q_0,\vec{q}{=}0)$ has been expanded around $q_0^2 = 0$ as $\Pi^S(q_0,0) = \Pi^S(0,0) + \frac{\partial\Pi^S}{\partial q_0^2}q_0^2$. By doing so, one can immediately read off the following relations consistent with the PCAC–Ward identities:

$$\gamma(\rho) = \frac{1-\frac{2c_3\rho}{f_\pi^2}}{(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2})^2}\,, \tag{14}$$

$$1-\frac{\partial\Pi^S}{\partial q_0^2} = \frac{1+\frac{2(c_2+c_3)\rho}{f_\pi^2}}{(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2})^2} \approx \frac{1}{1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}}\,, \tag{15}$$

$$1+\frac{\Pi^S(0,0)}{m_\pi^2} = \frac{1}{1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}}\,, \tag{16}$$

where in eq. (15) the following approximate relation has been used: $2(c_2+c_3)m_\pi^2 \approx -\sigma_N$. Note that $(1-\frac{\partial\Pi^S}{\partial q_0^2})^{-1} = (g_\pi^*(\rho)/g_\pi)^2$ as expected. In approximating

$$\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)^{-1} \approx 1+\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\,, \tag{17}$$

and inserting into eqs. (15-16) the standard (original) parametrizations are recovered:

$$\Pi^S(0,0) \approx \frac{\sigma_N\rho}{f_\pi^2} \approx -\frac{\partial\Pi^S}{\partial q_0^2}\,. \tag{18}$$

The relations (14–16) show that Migdal's pion field has to be identified with $\widetilde{\pi}(\rho)$ from eq. (12).

III. THE QUARK CONDENSATE WITHIN A FINITE FERMI SYSTEM

One possibility for evaluating the GOR relation on the composite hadron level is to assume validity of *PCAC as an operator relation*, $\partial^\mu \widehat{A}_\mu^a = f_\pi m_\pi^2 \widehat{\pi}^a$, and to exploit the Migdal propagator (under the identifications (14–16) now shown to be consistent with PCAC) for the in–medium pion. In doing so, the following relation is obtained [7]:

$$\begin{aligned}\Pi^{\rm GOR} &= \lim_{q_0\to 0,\vec{q}\to 0}\frac{i}{3}\int d^4x\, e^{-iq\cdot x}\langle\,\widetilde{0}\mid T\partial^\mu \widehat{A}_\mu^a(x)\,\partial^\nu \widehat{A}_\nu^a(0)\mid\widetilde{0}\,\rangle \\ &= \lim_{q_0\to 0,\vec{q}\to 0}\frac{i}{3}f_\pi^2 m_\pi^4\int d^4x\, e^{-iq\cdot x}\langle\,\widetilde{0}\mid T\widehat{\pi}^a(x)\,\widehat{\pi}^a(0)\mid\widetilde{0}\,\rangle \\ &= \lim_{q_0\to 0}\frac{f_\pi^2 m_\pi^4}{\left(1-\frac{\partial\Pi^S}{\partial q_0^2}\right)q_0^2 - m_\pi^2\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right)} = -\frac{f_\pi^2 m_\pi^2}{1+\frac{\Pi^S(0,0)}{m_\pi^2}}\,.\end{aligned} \tag{19}$$

On the other side, in the $\vec{q} \to 0$ limit PCAC is determined only by the partial time derivative of the axial charge density which can be replaced by

$$\partial^\mu \widehat{A}^a_\mu \xrightarrow{\vec{q}\to 0} \partial^0 \widehat{A}^a_0 = i[\mathcal{H}, \widehat{A}^a_0]\,, \tag{20}$$

where $\mathcal{H}$ stands for the QCD hamiltonian density. By integrating by parts one can then cast the GOR correlator into the form

$$\Pi^{\mathrm{GOR}} = \frac{1}{3}\Sigma^3_{a=1}\langle\, \widetilde{0} \mid [\widehat{A}^a_0, [\widehat{A}^a_0, \mathcal{H}]] \mid \widetilde{0}\,\rangle\,. \tag{21}$$

As long as the chiral symmetry violating term in the QCD lagrangian corresponds to the non–zero current quark masses, Π^{GOR} can be expressed in terms of the averaged current quark mass m_q and the quark condensate at non–zero density $\langle\, \widetilde{0} \mid \bar{q}q \mid \widetilde{0}\,\rangle$ according to $\Pi^{\mathrm{GOR}} = 2m_q\langle\, \widetilde{0} \mid \bar{q}q \mid \widetilde{0}\,\rangle$, where the flavor independence of the condensate has been assumed. Together with eqs. (19) and the PCAC-consistent identification (16) the last equation leads to the following results

$$2m_q\langle\, \widetilde{0} \mid \bar{q}q \mid \widetilde{0}\,\rangle = -\frac{f_\pi^2 m_\pi^2}{1 + \frac{\Pi^S(0,0)}{m_\pi^2}} \tag{22}$$

$$= -\left(1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2}\right) f_\pi^2 m_\pi^2 = -\left(f^t_\pi(\rho)\right)^2 (m^*_\pi)^2\ . \tag{23}$$

In this way (with the help of the PCAC-consistent relation (16)) eq. (11) is exactly reproduced – in the PCAC scheme on the operator level. If the approximations from eq. (18) were inserted for $\Pi^S(0,0)$ into eq. (22) a seemingly hyperbolic decrease of the quark condensate with density would be obtained in line with the result reported in [6]. The reason for this incorrect interpretation of eq. (22) is an inconsistent treatment of PCAC on the quark level. The assumption of the validity of PCAC on the operator level for composite hadrons should not be combined with the approximations of eq.(18) which do destroy the form of the Migdal propagator as developed in consistency with PCAC on the current quark level. Note that there is no compelling reason to derive the GOR relation by exploiting the PCAC hypothesis. If, however, a definite scheme as e.g. PCAC has once been chosen, then all the quantities have to be calculated only in that very scheme.

A *second possibility*, to derive the density dependence of the quark condensate within Migdal's theory of the finite Fermi systems is to calculate the PCAC condition on the *matrix element level* rather than postulating its validity as operator relation. This is much closer than the first approach to the generating functional approach which correlates the *vacuum expectation values* of time-ordered products of field-operators. The effect of the polarization of the medium during the propagation of a $S-$ / $P-$wave pion on the respective pion weak decay constants (in turn denoted by $(f_\pi^S)_{\rm FFS}$ and $(f_\pi^P)_{\rm FFS}$) is expressed by means of the following parametrizations of the matrix elements of the pion weak axial vector current $\widehat{A}^a_{\mu(\pi)}$:

$$\langle\, \widetilde{0} \mid \vec{\widehat{A}}{}^a_{(\pi)} \mid \widetilde{\pi}^b \,\rangle =: (f_\pi^P)_{\rm FFS}(\rho)\; i\vec{q}\; \delta^{ab}\,, \tag{24}$$

$$\langle\, \widetilde{0} \mid \widehat{A}^a_{0(\pi)} \mid \widetilde{\pi}^b \,\rangle =: (f_\pi^S)_{\rm FFS}(\rho)\; iq_0\, \delta^{ab}\,, \tag{25}$$

$$\langle\, \widetilde{N}(\vec{p}_2) \mid \vec{\widehat{A}}{}^a_{(\pi)} \mid \widetilde{N}(\vec{p}_1) \,\rangle =: i\vec{q}\,(f_\pi^P)_{\rm FFS}(\rho)\, \langle\, \widetilde{N}(\vec{p}_2) \mid \widehat{\pi}^a \mid \widetilde{N}(\vec{p}_1) \,\rangle\,, \tag{26}$$

$$\langle\, \widetilde{N}(\vec{p}_2) \mid \widehat{A}^a_{0(\pi)} \mid \widetilde{N}(\vec{p}_1) \,\rangle =: iq_0\,(f_\pi^S)_{\rm FFS}(\rho)\, \langle\, \widetilde{N}(\vec{p}_2) \mid \widehat{\pi}^a \mid \widetilde{N}(\vec{p}_1) \,\rangle\,, \tag{27}$$

where $q^\mu = p_1^\mu - p_2^\mu$. The constants $(f_\pi^S)_{\rm FFS}(\rho)$ and $(f_\pi^P)_{\rm FFS}(\rho)$ are then calculated [17] from the requirement (the so–called generalised Goldberger–Treimann (GT) relation) on the proportionality of the matrix elements of in–medium pion source operator (denoted by $\langle\, \widetilde{N}(\vec{p}_2) \mid \widehat{J}^a_\pi \mid \widetilde{N}(\vec{p}_1) \,\rangle$) to that of the divergence of the purely nucleonic axial current operator $\widehat{A}^a_{\mu(N)}$,

$$\lim_{m_\pi^2(1+\frac{\Pi^S(0,0)}{m_\pi^2})\to 0} -iq^\mu \langle \widetilde{N}(\vec{p}_2)|\widehat{A}^a_{\mu(N)}|\widetilde{N}(\vec{p}_1)\rangle \;=\; f_\pi^{\rm AP}(\rho)\langle \widetilde{N}(\vec{p}_2)|\widehat{J}^a_\pi|\widetilde{N}(\vec{p}_1)\rangle\,, \tag{28}$$

$$\widehat{A}^a_{\mu(N)} := -\bar{\Psi}_N\gamma_\mu\gamma_5\frac{\tau^a}{2}\Psi_N \quad \text{and} \quad (\Delta_\pi)^{-1}_{\rm FFS}\,\widehat{\pi}^a = -\widehat{J}^a_\pi\,, \tag{29}$$

where Ψ_N stands for the nucleon field operator and $f_\pi^{\rm AP}(\rho)$ satisfies the relation

$$\sqrt{2} f_\pi^{\rm AP}(\rho) = \lim_{m_\pi^2(1+\frac{\Pi^S(0,0)}{m_\pi^2})\to 0} (\Delta_\pi)_{\rm FFS}\,(-iq^\mu)\langle\, \widetilde{0} \mid \widehat{A}^\pm_{\mu(\pi)} \mid \widetilde{\pi}^\pm \,\rangle\,. \tag{30}$$

The latter equation is consistent with the definition of the bare pion weak decay coupling at zero density through the condition

$$\sqrt{2} f_\pi = \lim_{m_\pi^2 \to 0} \left(q^2 - m_\pi^2\right)^{-1} (-iq^\mu) \langle\, 0 \mid \widehat{A}^{\pm}_{\mu(\pi)} \mid \pi^\pm \,\rangle \,. \tag{31}$$

The limit $m_\pi^2 \to 0$ for the vacuum is the justification for the corresponding limit in eq. (30). The r.h.s. of eq. (30) is related to the axial charge – pseudoscalar (AP) two–point function [8]

$$\frac{\delta^2 Z_{\rm MF}}{\delta A_0^a(-q)\,\delta j^b(q)}|_{A_\mu=V_\mu=j=0,s=\mathcal{M}} = iq^0 \frac{f_\pi \left(1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2}\right)\delta^{ab}}{q_0^2 - \vec{q}^{\,2}\frac{1-D^{ii}\rho/f_\pi^2}{1+D^{00}\rho/f_\pi^2} - m_\pi^{*\,2}} + \mathcal{O}(m_\pi)\,, \tag{32}$$

via

$$f_\pi^{\rm AP}(\rho)\delta_{ab} = \lim_{\vec{q}\to 0; m_\pi^{*\,2}\to 0} -iq_\mu \frac{\delta^2 Z_{\rm MF}}{\delta A_\mu^a(-q)\,\delta j^b(q)}|_{A_\mu=V_\mu=j=0,s=\mathcal{M}} = f_\pi \left(1 - \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2}\right)\delta_{ab}\,. \tag{33}$$

Now in inserting eqs. (13), (24) and (25) into eq. (30) the following solution for $(f_\pi^S)_{\rm FFS}$ (see [18] for more details) is obtained:

$$(f_\pi^S)_{\rm FFS}(\rho) = f_\pi^{\rm AP}(\rho)\left(1 - \frac{\partial \Pi^S}{\partial q_0^2}\right) = f_\pi \left(1 - \frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)\left(1 - \frac{\partial \Pi^S}{\partial q_0^2}\right)\,. \tag{34}$$

The factor $(1 - \frac{\partial \Pi^S}{\partial q_0^2})$ accounts for the kinetic term of the S–wave pion self energy. Note that already the definition (25) implies that $(f_\pi^S)_{\rm FFS}(\rho)$ has to be interpreted as the weak axial vector decay constant of *Migdal's pion* (12). It can of course be constructed directly from the kinetic term of the mean-field lagrangian (4), more precisely from the A_μ^a–π^a interaction term where the axial vector source, A_μ^a, should not be mixed up with the hadronic axial current $\widehat{A}^a_{\mu(\pi)}$:

$$A_0^a\, iq^0\, f_\pi \left(1 + \frac{D^{00}\rho}{f_\pi^2}\right)\pi^a = A_0^a\, iq^0 \left\{ f_\pi \frac{1 + \frac{D^{00}\rho}{f_\pi^2}}{1 - \frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}}\right\}\widetilde{\pi}^a = A_0^a\, iq^0\, (f_\pi^S)_{\rm FFS}(\rho)\, \widetilde{\pi}^a \tag{35}$$

in agreement with eq. (34) — in case eq.(15) has been inserted. If Migdal's pion field in eq.(35) is once more renormalized (multiplied) by the factor $g_\pi/g_\pi^*(\rho) = (1 - \frac{\partial\Pi^S}{\partial q_0^2})^{1/2}$, such that the corresponding inverse pion-propagator has weight one relative to the time-like ∂_0^2 term, the physical in-medium pion decay constant $f_\pi^t(\rho)$ can be read from eq.(35) as

$$A_0^a\, iq^0\, f_\pi \left(1 + \frac{D^{00}\rho}{f_\pi^2}\right)\pi^a = A_0^a\, iq^0\, f_\pi^t(\rho)\, \left(\frac{g_\pi}{g_\pi^*(\rho)}\widetilde{\pi}^a(\rho)\right) \tag{36}$$

in agreement with eq.(5). Note that the scheme-dependence cancels out from the combination $(\widetilde{\pi}^a(\rho)g_\pi/g^*_\pi(\rho))$, such that the prefactor $f^t_\pi(\rho)$ is scheme-independent, too, as it should.

The PCAC relation within Migdal's theory is then obtained in evaluating the matrix element of the divergence of the summed ($\widehat{A}^a_\mu = \widehat{A}^a_{\mu(N)} + \widehat{A}^a_{\mu(\pi)}$) nucleon and pion weak axial current between in–medium nucleon states by means of eqs. (24–27) in the rest frame of the pion according to

$$\begin{aligned}
&\lim_{\vec{q}\to 0}\,(-iq^\mu)\,\langle\,\widetilde{N}(\vec{p}_2)\mid \widehat{A}^a_{\mu(N)} + \widehat{A}^a_{\mu(\pi)}\mid \widetilde{N}(\vec{p}_1)\,\rangle \\
&= \lim_{\vec{q}\to 0}\,(-iq^\mu)\langle\,\widetilde{N}(\vec{p}_2)\mid \widehat{A}^a_{\mu(N)}\mid \widetilde{N}(\vec{p}_1)\,\rangle \\
&\quad + (f^S_\pi)_{\rm FFS}\frac{q_0^2\langle\,\widetilde{N}(\vec{p}_2)\mid -\widetilde{J}^a_\pi\mid \widetilde{N}(\vec{p}_1)\,\rangle}{\left(1-\frac{\partial\Pi^S}{\partial q_0^2}\right)q_0^2 - m_\pi^2\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right)}\,.
\end{aligned} \tag{37}$$

In inserting into the last equation the value for $(f^S_\pi)_{\rm FFS}$ from eq. (34) and in accounting for eq. (28) as well as for definition of the pion field via eq. (29), one obtains the final form of the PCAC relation in Migdal's theory as

$$\begin{aligned}
&-iq^\mu\langle\,\widetilde{N}(\vec{p}_2)\mid \widehat{A}^\pm_{\mu(N)} + \widehat{A}^\pm_{\mu(\pi)}\mid \widetilde{N}(\vec{p}_1)\,\rangle \\
&= \sqrt{2}f_\pi\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)m_\pi^2\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right)\langle\,\widetilde{N}(\vec{p}_2)\mid \widehat{\pi}^\pm\mid \widetilde{N}(\vec{p}_1)\,\rangle \\
&= \sqrt{2}f_\pi m_\pi^2\langle\,\widetilde{N}(\vec{p}_2)\mid \widehat{\pi}^\pm\mid \widetilde{N}(\vec{p}_1)\,\rangle\,.
\end{aligned} \tag{38}$$

The PCAC-consistent relation (16) has been used for the last step. Eq. (38) in fact means validity of PCAC as operator relation within a finite Fermi system and thus allows the evaluation of the GOR correlator within Migdal's approach along the line of eq. (19). As a result, also in Migdal's treatment of a dense medium the linear decrease of the quark condensate with density is recovered,

$$-\,2m_q\langle\,\widetilde{0}\mid \bar{q}q\mid \widetilde{0}\,\rangle = f_\pi^2 m_\pi^2\,\frac{1}{1+\frac{\Pi^S(0,0)}{m_\pi^2}} = f_\pi^2 m_\pi^2\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right) = \left(f^t_\pi(\rho)\right)^2(m^*_\pi)^2\,. \tag{39}$$

Thus again the factor $1/1+\frac{\Pi^S(0,0)}{m_\pi^2}$ is converted back to the rescaling factor of the pion field at zero matter density π^a to the Migdal field $\widetilde{\pi}^a$ at finite matter density from eq. (12) — in

agreement with the generating functional formalism [14]. In this way the QCD predicted linear decrease of the quark condensate in the medium is, as expected, recovered.

IV. SUMMARY AND DISCUSSION

To summarize we wish to stress that it is the quark condensate that depends linearly on density rather than the S–wave pion polarization function $\Pi^S(q_0, \vec{q}=0)$ as currently used in the literature. The quantity $\Pi^S(q_0, \vec{q}=0)$ in fact contains higher powers of ρ resulting from the expansions (15, 16)

$$\frac{\Pi^S(0,0)}{m_\pi^2} = \frac{1}{1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}} - 1 \;=\; \frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\left(1 + \frac{\sigma_N\rho}{f_\pi^2 m_\pi^2} + \left(\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)^2 + \cdots\right) \approx -\frac{\partial \Pi^S}{\partial q_0}\Big|_{q_0^2=0} . \quad (40)$$

In an early work ref. [17] the quantity $f_\pi^{\mathrm{AP}}(\rho)$ in eq. (30) was approximated by the bare pion weak decay constant f_π. This approximation implies the absence of the factor $1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}$ in the expression for $(f_\pi^S)_{\mathrm{FFS}}(\rho)$ in eq. (34) and leads therefore to the following changes in the PCAC condition of eq. (38):

$$\begin{aligned} & f_\pi m_\pi^2\left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right)\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right) = f_\pi m_\pi^2 \\ \Longrightarrow\; & f_\pi m_\pi^2\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right) = f_\pi m_\pi^2 \frac{1}{1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}} . \end{aligned} \quad (41)$$

With that, the quark condensate resulting from the evaluation of the GOR correlator (19) in that approximation would be proportional to

$$-\,2m_q\langle\,\widetilde{0}\mid\bar{q}q\mid\widetilde{0}\,\rangle \;''{=}''\; f_\pi^2 m_\pi^2\left(1+\frac{\Pi^S(0,0)}{m_\pi^2}\right) = f_\pi^2 m_\pi^2 \Big/ \left(1-\frac{\sigma_N\rho}{f_\pi^2 m_\pi^2}\right) , \quad (42)$$

and thus increasing instead of decreasing with density. This observation underlines once more the necessity for a careful and consistent treatment of PCAC before exploiting it for the calculation of measurable and thus model– or scheme–independent QCD quantities. There is no real need to refer to PCAC on the composite hadron level to calculate quantities on the current quark level. However, if one prefers to work in this scheme, one has to apply it without violating consistency relations having their roots in the current quark dynamics.

As presented above, the big virtue of the generating functional scheme is that it allows for a transcription of the source structures from the current quark level of the underlying QCD to the effective hadron level without invoking any model assumptions of how the composite hadrons are built up from the quarks.

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